Chem 442 Challenge Problem No. 9.

Problem 9-8 essentially asked whether a set of four distinct natural numbers a, b, c, and d exists that satisfies

$$a^2 + b^2 = c^2 + d^2. \tag{1}$$

We could locate $(a, b, c, d) = (1, 7, 5, 5)$ as an earliest example by a blanket search. Since this combination corresponds to a three-fold degeneracy of the particle-in-a-well problem, we can argue that a spectroscopic measurement of the particle in a well serves as a quantum computer for detecting the roots of Eq. (1) (the emission band intensity will be proportional to the degree of degeneracy).

Fermat's Last Theorem states that there are no triplet of natural numbers a, b, and c that satisfies

$$a^n + b^n = c^n, \tag{2}$$

when $n \geq 3$. Although this has been proven by Andrew Wiles at University of Oxford, can you conceive of a quantum-mechanical system whose energy eigenstates have the degree of degeneracy dictated by the above formula, so that one can use that system as a quantum computer for a blanket search of a counterexample of this Theorem (which however does not exist).

There are many other number theory conjectures, which classical computers play important roles in analyzing. Can you devise a quantum computer that exploits the degeneracy condition in order to perform a blanket search of a counterexample of any of these conjectures?